Probabilistic Estimates of Vulnerability to Explosive Overpressures and Impulses

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Abstract

A probabilistic risk assessment procedure is followed to estimate the probability of protection at facilities from vehicle-borne explosions. Using truncated normal distributions for both the TNT equivalencies of the charge and the standoff distances, a probability for protection is calculated for a single degree of freedom (SDOF) blast protection design curve. Numerical examples are given for two representative design curves that give probabilities of protection of 80% and 91%.

1. Introduction

Different scenarios can be envisioned that would be relevant to the design of terrorist mitigation measures at a facility. For explosive threats, one could assume a scenario in which the standoff distance is the minimum possible distance to the intended target, produces collateral damage to nearby facilities and that the explosive source is very large. For example, one such scenario might be 40,000 lb of an efficiently exploded ammonium nitrate/fuel oil mixture contained within an 18-wheeler. A related scenario might be where the 18-wheeler is located as close as physically possible to the intended target or where the size of the explosive source is not as large. If a smaller vehicle is envisioned to transport the source, 40,000 lb would be an overestimate of the explosive payload. In addition, the TNT-equivalent mass of the explosive can vary depending on the physical and chemical details of the explosive mixture.

The uncertainties in explosive strength and standoff distance suggest that a statistical assessment of the threat parameters could play a role in the mitigation design. This type of approach has been proffered recently by Stewart, Netherton, and Rosowsky (2006). In this reference they argue that there is so much uncertainty associated with terrorism that an attempt should be made to quantify the uncertainties, and that these results should then be used in established probabilistic risk assessment procedures to systematically assess the viability and relative benefits of different mitigation measures.

Netherton and Stewart (2009) considered the variability in explosive blast loading caused by a multiplicity of factors with respect to façade glazing, and used Monte Carlo simulation to calculate probabilities of glazing damage and safety hazards conditional on scenario-driven parameters.

The present paper also provides a statistical assessment of an explosive-borne threat to window structures in facilities. However, we adopt an exclusively analytic approach based on published scaling relations for the key explosive parameters, impulse and overpressure, as well as the design specifications of the window. Although it does not take into consideration every physical parameter that affects explosive impact (most notably the presence of intervening structures), this method obviates the need for computer simulation and yields a simple if approximate method to assess explosive risk.

Specifically, we estimate the probability of protection one might expect against likely blast overpressures and impulses. This is based on both the probability function for explosive strength and standoff distance and the design curve used for constructing components of a curtain wall façade, a popular form of construction for modern office facilities. In this paper, the design curve is derived from a simple single degree of freedom (SDOF) component model.

In addition, the method of assessing the probability of protection afforded by a specific building component as described herein can be generalized and applied to other security scenarios for which scaling relations exist for relevant physical parameters. This offers a potentially powerful tool in quantitatively assessing risk for a variety of scenarios.

Section 2 describes the probability distributions to be assumed for the TNT equivalence strength of the explosive source, and for the standoff distance between the explosive source and a potential target.

Section 3 discusses the blast parameters (overpressure and impulse) that determine the damage experienced by a structure.

Section 4 summarizes scaling laws that relate the blast overpressure and impulse to the explosive strength and standoff distance.

Section 5 then discusses a blast protection design curve that is based on a SDOF model and on data provided by a consulting firm for a specific window design.

Section 6 derives a specific blast mitigation probability of protection using the probability function and the design curve.

Section 7 discusses the specific results and generalizes the approach to other operational risk problems.

2. Probability distributions for explosive source strength and for standoff distance

Based on past terrorist explosive attacks on structures, we shall assume that the explosive source is an ammonium nitrate/fuel oil (ANFO) mixture, and that this explosive mixture is delivered to the near vicinity of a facility by a vehicle. Uncertainties exist as to both the TNT equivalency of the ANFO, and to how close the vehicle will be to a facility when the mixture is detonated.

2a. Uncertainty in TNT equivalency factor

The construction and the efficacy of the resulting bomb have been widely discussed in the open literature. Two readily available on-line summary articles are:

> Explosives – ANFO *Ammonium nitrate-fuel oil), at http://www.globalsecurity.org/military/system/munition/explosives-anfo.htm
>
> ANFO, at http://en.wikipedia.org/wiki/ANFO

Differences in the preparation of ANFO explosive as well as the physical make-up of its components can introduce variability in TNT equivalency. These can be manifest as variability in the amount of absorbed water in the mixture, a lack of uniformity in mixing, differences in specific gravity, and the particular oxidizer and type of fuel oil used.

Because of the inherent variability in one or more of these factors, there is uncertainty as to the TNT equivalent mass that should be assumed for a terrorist attack. To provide a framework for assessing the blast parameters, we shall assume in the following that the probability $F_m(m)dm$ that a terrorist explosive attack will involve an explosive of TNT equivalent mass m in the interval (m, m+dm) has the shape of a normal probability distribution function with a mean m_o and a dispersion δm, We shall also assume that the truncated normal probability distribution is only nonzero for m larger than some minimum m_{min} and for m less than some maximum m_{max}.

Specifically,

$$F_m(m)dm = \begin{cases} C_m \exp[-(m-m_o)^2/(\delta m)^2]dm & \text{for } m_{min} < m < m_{max} \\ 0 \quad \text{otherwise} & \end{cases} \qquad [1a]$$

where the normalization constant is

$$C_m = (2/\pi^{1/2}\delta m)\,[\mathrm{Erf}((m_{max}-m_o)/\delta m) + \mathrm{Erf}((m_o-m_{min})/\delta m) \qquad [1b]$$

From the foregoing discussion, we shall use as a numerical example

$m_o = 32{,}000$ lbs TNT
$m_{min} = 12{,}000$ lbs TNT
$m_{max} = 64{,}000$ lbs TNT
$\delta m = 20{,}000$ lbs TNT [2]

The mean value, 32,000 lbs, corresponds to 40,000 lb of ANFO with a TNT equivalency factor of 0.8, whereas the minimum and maximum values, 12,000 and 64,000 lbs, correspond to 40,000 lb of ANFO with TNT equivalency factors of 0.3 and 1.6, respectively. The dispersion $\delta m = 20{,}000$ lbs has been set equal to $m_o - m_{min}$ to describe a likely spread in values comparable to the difference between the nominal 0.8 equivalency factor and the minimum equivalency factor of 0.3. This spread has been chosen rather than a spread equal to the difference between the maximum equivalency factor of 1.6 and the nominal factor of 0.8, to weight more heavily the multitude of factors that can decrease the TNT equivalency.

The truncated normal probability distribution $F_m(m)$ is displayed in Figure 1 for the numerical parameters of eq. [2].

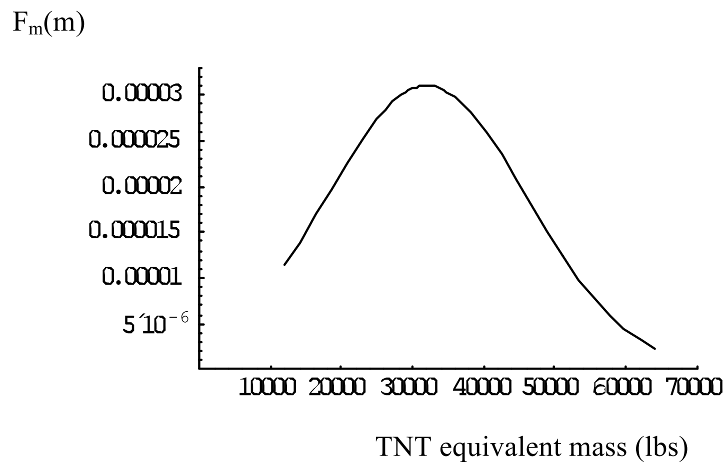


Figure 1. Probability function $F_m(m)$ for TNT equivalent mass m for the representative parameters of eq. [2].

2b. Uncertainty in Standoff Distance

Vehicle-borne explosive attacks represent a significant security concern and collateral damage experienced as a result of attacks against nearby facilities is a concern as well. This adds to the uncertainty associated with urban terrorist threats and thereby affects mitigation strategies under consideration. Strategically placed bollards will prevent a vehicle from ramming a facility as well as set a minimum vehicle access distance. We assume the closest distance of approach to a facility and still be within the iconic “orbit” of its intended target is 275 feet. Therefore, the standoff distance for a vehicle-borne explosive source should equal or exceed a 275 foot minimum.

However, a number of vehicle positions that border a target facility are equally likely from a terrorist’s perspective since any point along this border would inflict approximately the same damage to the target facility. However, differences in this distance could have a profound effect on collateral damage to neighboring facilities. The range of likely detonation positions relative to potential explosive damage to a neighboring facility is assumed to be principally dictated by the length of a street bordering the target building

Without further details of the terrorists' plans, we again resort to the normal probability distribution function as the one that best reflects the inherent uncertainty in key physical parameters. It should be noted that although a truncated normal distribution is most appropriate in this instance, the approach specified herein does not depend on the specific probability distribution. Since $F_m(m)$ and $F_r(r)$ are assumed to be independent, the joint probability and resulting contour plot could be calculated in similar fashion using alternative distribution functions. However, the dispersion associated with the chosen probability distributions which characterizes the uncertainty in potential threat scenarios is considered most important in calculating the probability of protection afforded by a given window design.

Specifically, we assume that the probability $F_r(r)dr$ that the standoff distance is between r and r+dr is also given by a truncated normal distribution:

$$F_r(r)dr = \begin{cases} C_r \exp[-(r-r_o)^2/(\delta r)^2]dr & \text{for } r_{min} < r < r_{max} \\ 0 \text{ otherwise} & \end{cases} \qquad [3a]$$

where the normalization constant is

$$C_r = (2/\pi^{1/2}\delta r)\ [\mathrm{Erf}((r_{max}-r_o)/\delta r) + \mathrm{Erf}((r_o-r_{min})/\delta r) \qquad [3b]$$

Here r_o denotes the value of the standoff distance at which the probability is largest: for example, when the detailed geometry of a target facility placement becomes available, this could be determined from the location of a vehicle that would place it as close as possible to the target. The dispersion δr measures the spread in likely values for the standoff distance. And as discussed earlier, r_{min} denotes the closest possible distance of approach to a neighboring facility and r_{max} denotes the maximum standoff distance from the neighboring facility for which an explosive-laden vehicle could do substantial damage to the target building.

To illustrate the approach, in the following we shall present a numerical example for

$$\begin{aligned} &r_{min} = 275 \text{ feet} \\ &r_{max} = 525 \text{ feet} \\ &r_o = 0.5\,(r_{min}+r_{max}) = 400 \text{ feet)} \\ &\delta r = r_o - r_{min} = 125 \text{ feet} \end{aligned} \qquad [4]$$

As more detailed information becomes available, these numbers can be modified.

The probability function $F_r(r)$ is displayed in Figure 2 for the numerical parameters of eq. [4].

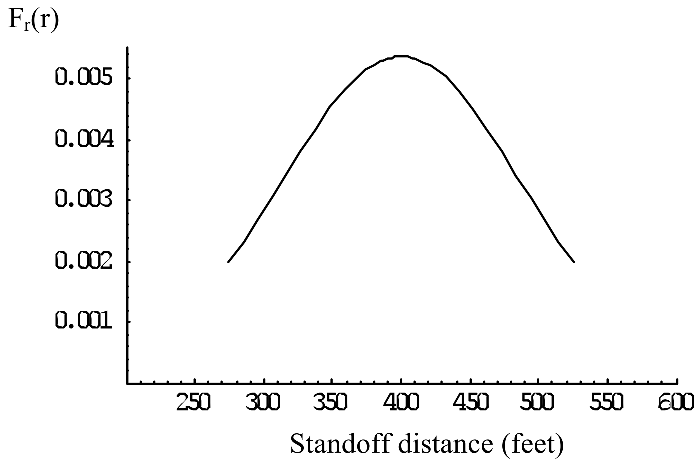


Figure 2. Probability function $F_r(r)$ for standoff distance r from a facility in proximity to the intended target for the representative parameters of eq. [4].

It is also of interest to look at the joint probability distribution function $F_{mr}(m,r)dmdr$ that gives the probability that the TNT equivalent mass m is in the range (m, m+dm) and the standoff distance is in the range (r,r+dr). Since we have assumed that the probability distribution functions of equations [1] and [2] are independent, we can write simply

$$F_{mr}(m,r) = F_m(m)\, F_r(r) \qquad [5]$$

The joint probability function $F_{mr}(m,r)$ is displayed in Figure 3 for the numerical parameters of eqs. [2] and [4]. As expected, the joint probability distribution function displays a single maximum at (m_o, r_o) with a spread in the TNT-equivalent mass m determined by δm and a spread in the standoff distance r determined by δr.

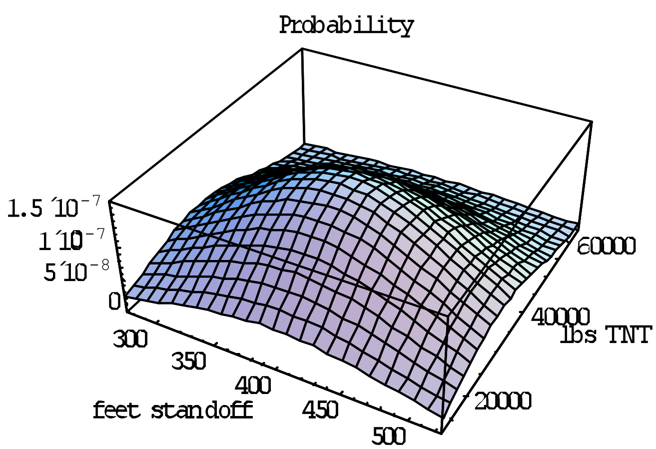
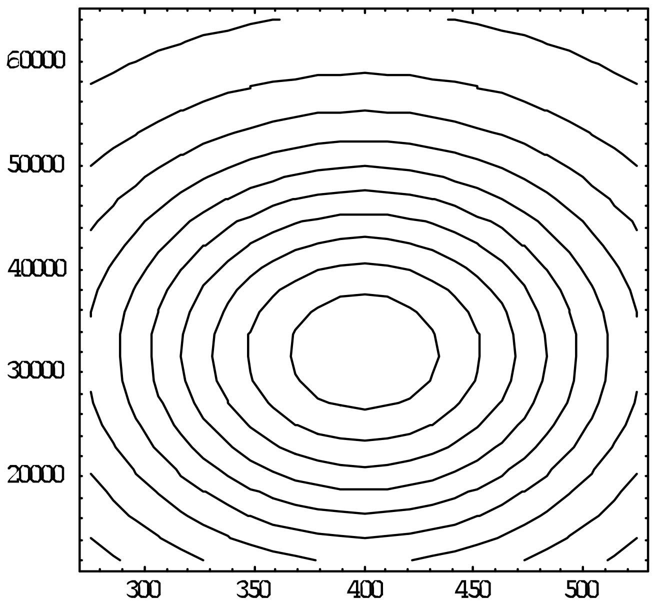


Figure 3 Joint probability function $F_{mr}(m,r)$ for the numerical parameters in eqs. [2] and [4]. In the 3D image of Figure 3a, the x-axis displays the TNT equivalent mass in lbs, and the y-axis displays the standoff distance in feet. In the contour plot of Figure 3b, the TNT equivalent mass is shown on the vertical axis and the standoff distance is shown on the horizontal axis.

The contour diagram of the probability function is shown in Figure 4.

TNT equivalent
mass m (lbs)



Standoff distance r (feet)

Figure 4. Contour plot of joint probability function $F_{mr}(m,r)$ for the numerical parameters in eqs. [2] and [4]. The TNT equivalent mass is shown on the vertical axis and the standoff distance is shown on the horizontal axis.

Finally, we note that a joint probability distribution could be calculated in this way for an unlimited number of physical parameters with an associated dispersion specified for each, assuming such parameters were independent. In this report, standoff distance and TNT equivalent mass were used since these directly affect overpressure and impulse as indicated in referenced scaling relations.

3. Structural damage from an explosive blast

In the preceding section we have discussed the probability distribution function $F_{mr}(m,r)dmdr$ for the TNT-equivalent mass m and the standoff distance r. The equivalent mass m and standoff distance r are not the explosive blast variables that are most directly related to structural damage. Rather, these variables are the blast overpressure p and the blast impulse I [See, e.g., M. Held, 1983]. In Section 4 we shall discuss how p=p(m,r) and i=i(m,r) are related to m and r. In this section, we shall briefly review why it is that blast overpressure p and blast impulse i are the quantities of most direct interest to determining structural damage.

The interaction of a blast wave with a structure is quite a complicated phenomenon. Ngo, Mendis, Gupta, and Ramsay (2007) have provided a recent overview of the problem.

At any point in the path of an explosive blast wave, the pressure rises rapidly to a maximum value that is known as the *overpressure*. This is followed by a slower – although still quite rapid - decay of the pressure to below the ambient pressure. During this decay to ambient pressure, an *impulse* is delivered to any object experiencing the increased pressure. This portion of the blast wave is then followed by a smaller drop in pressure below the ambient pressure and a more gradual return to atmospheric pressure. This portion can exert suction on an object, and can also deliver debris to the object that has been sucked into the blast wave.

In a crude analysis, the effect of a blast wave on a structure is related to the magnitude of the overpressure. Thus, in FEMA and DOD documents, tables similar to Table 1 are often displayed. Table 1, taken from the DOD document TB 700-2 (cited in Jeremic and Bajic, 2006), lists the types of damage to be expected from various blast overpressures.

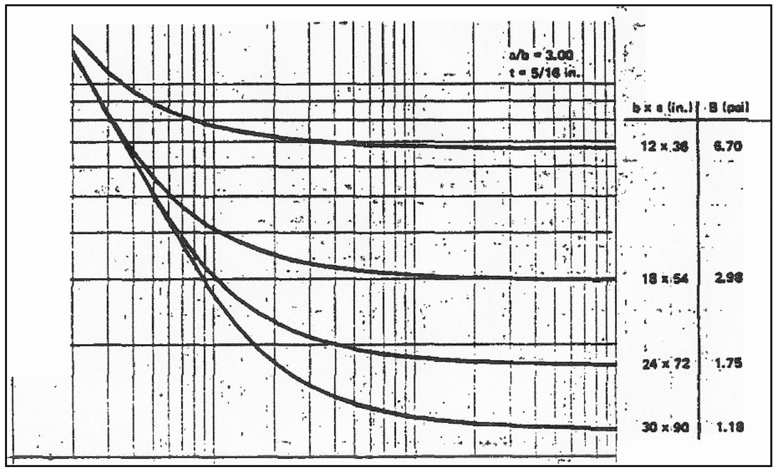
Table 1. Expected shock wave effects on objects (from DOD's TB 700-2, cited in Jeremic and Bajic, 2006). - psi units added

No.	Overpressure		Expected damage
	kPa	psi	
1	1.0-1.5	0.15-0.22	Window glass cracks
2	3.5-7.6	0.51-1.1	Minor damage in some buildings
3.	7.6-12.4	1.1-1.8	Metal panels deformed
4.	12.4-20	1.8-2.9	Concrete walls damage
5.	Over 35	Over 5.1	Wooden construction buildings demolition
6.	27.5-48	4.0 -7.0	Major damage on steel construction objects
7.	40-60	5.8-8.7	Heavy damage on reinforced concrete buildings
8.	70-80	10-11.6	Probable demolition of most buildings

However, the duration of the pressure pulse in the blast also plays a role in determining damage, i.e. the damage is related to the impulse as well as to the overpressure. An example of this is given in Figure 5, based on a figure from DOD's TM15-1300 (cited in Ngo, Mendix, Gupta, Ramsay, 2007). Figure 5 shows a design chart for a tempered glass panel. It shows clearly that the survival of a panel depends not just on the peak pressure but also on the blast impulse to which it will be subjected.

The reason why both overpressure and impulse are important in determining structure damage becomes evident in a simple model that is sometimes used to describe blast wave/structure interaction: In this model - the so-called single degree if freedom (SDOF) model - the structure (or an element of the structure) is replaced by an equivalent system of one concentrated mass and one weightless spring that represents the resistance of the structure to deformation.

Peak blast pressure
(units unspecified below)



1 10 100 1000
Duration of blast pressure (msec)

Figure 5. A design chart for a tempered glass panel (TM5-1300, Figure 6-43).
It displays the peak pressure that different panels can withstand as the blast duration is varied.

It is well known that the response of a mass on a spring to a transient force is qualitatively different when the natural oscillation period (determined by the spring constant and the mass) is greater than the force duration than when it is less than the force duration. In the former, the impulse (the integral of the force over time) determines the initial velocity of response of the mass, while in the latter the displacement of the mass is practically proportional to the force throughout its application. In a building, different elements of the building will have different natural oscillation frequencies, and the response of any particular element in its surroundings will respond accordingly either to the blast overpressure exerted on it or to the blast impulse.

In Ngo, Mendix, Gupta, Ramsay, 2007, a half dozen computer programs are listed that are used for calculating structural responses and for designing blast-resistant structures.

4. Scaling laws for blast overpressure and impulse

Several approximate scaling laws have been suggested in the literature to relate blast overpressure and impulse - the variables of interest for determining structural damage – to the TNT-equivalent mass and standoff distance of the explosive. Several examples are listed below:

Overpressure

- **M.A. Sadovski** [cited in Jeremic and Bajic, 2006]

  $$p(m,r) = 0.085(m^{1/3}/r) + 0.3(m^{2/3}/r^2) + 0.8(m/r^3) \text{ MPa} \qquad [6]$$

- **H.L. Brode (1955)**

  $p(m,r) = 0.1 + 0.67\ (m/r^3)$ MPa   for $p(m,r) > 1$ MPa   [7a]

  $p(m,r) = 0.0975(m^{1/3}/r) + 0.1455(m^{2/3}/r^2) + 0.585(m/r^3) - 0.0019$ MPa

  for $0.01 < p(m,r) < 1$ MPa   [7b]

- **C.A. Mills (1987)**

  $p(m,r) = 0.108(m^{1/3}/r) - 0.114(m^{2/3}/r^2) + 1.772(m/r^3)$ MPa   [8]

- **M. Held (1983)**

  $p(m,r) = 2(m^{2/3}/r^2)$ MPa   [9]

Impulse

- **M.A. Sadovski** [cited in Jeremic and Bajic, 2006]

  $i(m,r) = 200(m^{2/3}/r)$ Pa-s   [10]

- **M. Held (1983)**

$i(m,r) = 300(m^{2/3}/r)$ Pa-s   [11]

Note: In these expressions, the TNT-mass equivalent m is expressed in kg and the standoff distance r is expressed in meters

It is interesting to see how varied the scaling laws for overpressure are, and how not many scaling laws have been proposed for the impulse. This is a reflection of the complex nature of the explosive blast wave.

For example, the impulse depends strongly on the shape of the decaying pressure pulse following the large overpressure due to the blast shock wave. Multiplication of the overpressure by cited durations for the pressure pulse do not give good approximations for the impulse.

In the following we shall use Sadovski's expressions for both the overpressure and the impulse, i.e. we shall use eq. [6] for p(m,r) and eq. [10] for i[m,r]. Sadovski based his scaling laws on numerous experimental results; and in addition, for the range of overpressures of interest, comparisons of the shapes of actual data curves with curve fitting expressions seem to favor expressions containing multiple terms [See, e.g., G.F. Kinney (1962)].

Equations [6] and [10] are rewritten below in units of psi, psi-msec, lbs, and ft

$p(m,r) = 31.11(m^{1/3}/r) + 276.9(m^{2/3}/r^2) + 1863(m/r^3)$ psi   for m in lbs and r in ft   [6]

$i(m,r) = 56.25(m^{2/3}/r)$ psi msec   for m in lbs and r in ft   [10]

The blast overpressure p(m,r) given by eq. [6] as a function of TNT equivalent mass m and standoff distance r is displayed in Figure 6. Figure 7 shows the blast impulse i(m,r) given by eq. [10].

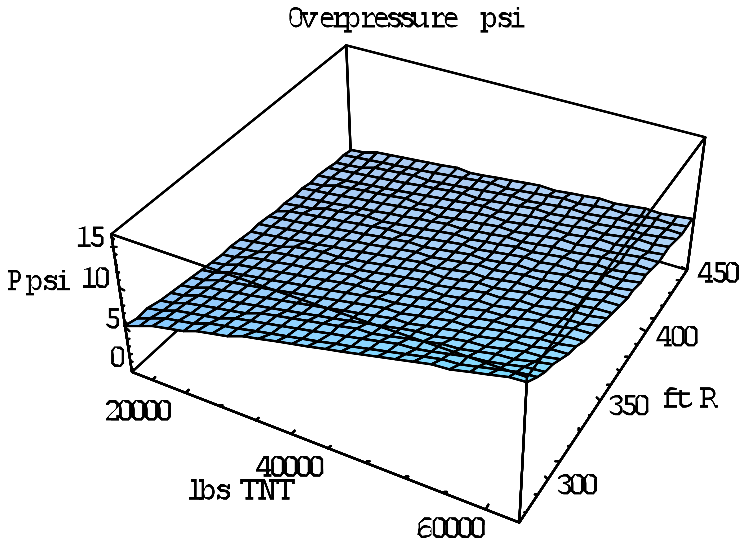


Figure 6. Blast overpressure p(m,r) as a function of TNT equivalent mass m and standoff distance r, from eq. [6]. The z-axis shows the overpressure p(m,r) in psi. The x-axis shows the TNT-equivalent mass m in lbs, and the y-axis shows the standoff distance in feet.

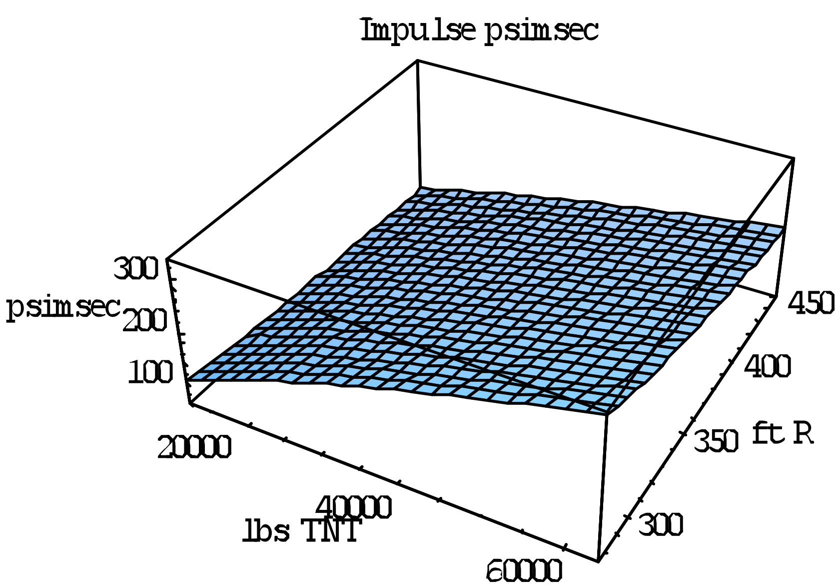


Figure 7. Blast impulse i(m,r) as a function of TNT equivalent mass m and standoff distance r, from eq. [10]. The z-axis shows the impulse i(m,r) in psi msec. The x-axis shows the TNT-equivalent mass m in lbs, and the y-axis shows the standoff distance in feet.

It is also of interest to solve eqs. [6] and [10] for TNT-equivalent mass m in terms of blast overpressure p and impulse i, as well as for standoff distance r in terms of p and i. The solutions for m(p,i) and r(p,i) are rather messy algebraically, but can be obtained straightforwardly with Mathematica symbolic algebra software. This yielded three possible roots each for m(p,i) and r(p,i) where two of the three are complex and therefore nonphysical.

The dependence of the standoff distance r(p,i) on overpressure p and impulse i is shown in Figure 8, whereas Figure 9 shows how the TNT-equivalent mass m(p,i) depends on p and i.

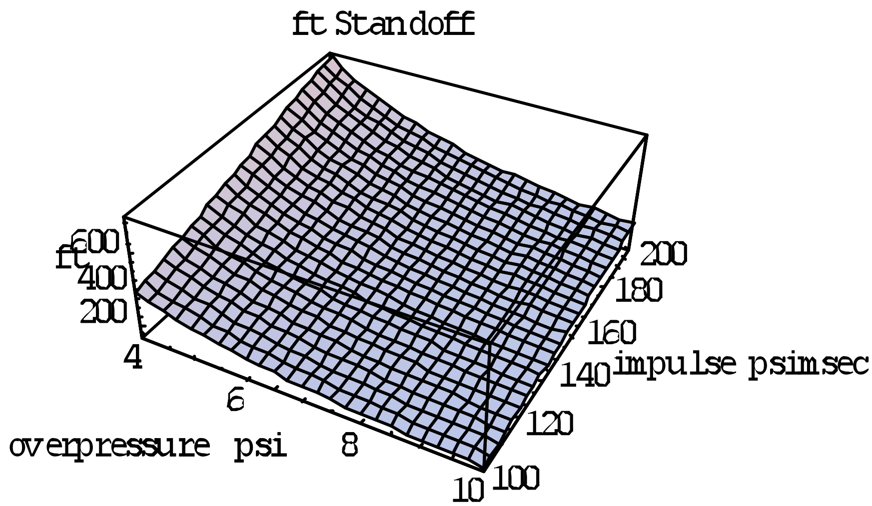


Figure 8. Standoff distance r as a function of overpressure p and impulse i. The standoff distance is shown along the z-direction in feet, with the overpressure shown along the x-axis in psi and the blast impulse shown along the y-axis in psi msec.

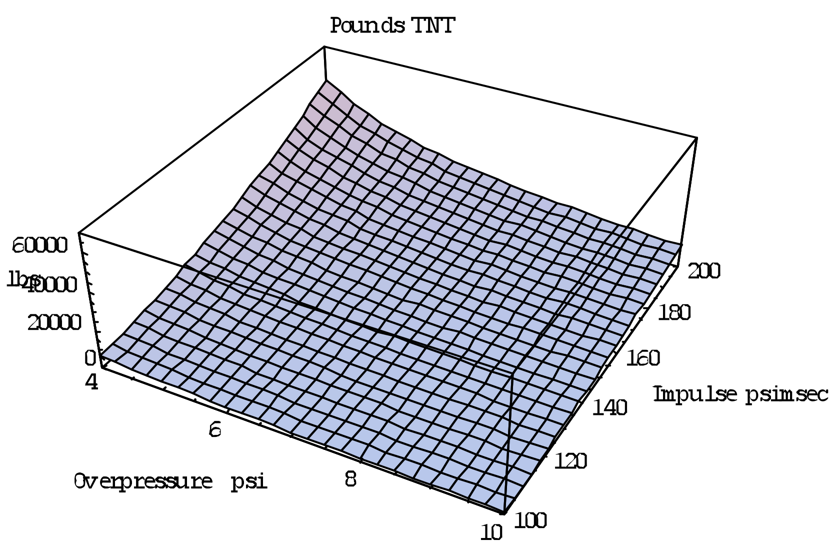


Figure 9. TNT-equivalent mass m as a function of overpressure p and impulse i. The TNT-equivalent mass is shown along the z-direction in lbs, with the overpressure shown along the x-axis in psi and the blast impulse shown along the y-axis in psi msec.

The plots show that the largest values for standoff distance r and TNT-equivalent mass m occur when the overpressure, p, is small and the impulse, i, is large.

5. Design curves for blast protection

In order to obtain an estimate for the degree of blast protection, the probability considerations need to be combined with a design curve that sets the limits of integration for the probability function.

Simple SDOF model For calculating the response of a structure such as a window to a blast wave, an approximation that is sometimes used is to replace the components of the structure by a simple mass M held in place by a spring of spring constant K. This approximation is termed the single degree of freedom (SDOF) approximation.

The displacement y(t) of the mass in response to a time dependent force F(t) is obtained from the equation

$$d^2y/dt^2 + \omega^2 y = F/M \qquad [11]$$

where the natural oscillation angular frequency ω is given by $\omega^2 = K/M$. If the force is due to a transient pressure pulse P, we can further write $F= PA$ where A is the area of the component.

Suppose we characterize the transient pressure pulse by the crude approximation

$$P(t) = \mathrm{P}\exp(-t/T) \qquad [12]$$

The impulse associated with this is I = PT
Then, assuming that the initial displacement and velocity of the component are zero, the solution is simply

$$y(t) = \alpha[\exp(-t/T) -\cos(\omega t) +(1/(\omega T\sin(\omega t))] \qquad [13]$$

where

$$\alpha = (PA/M)I^2/(P^2+(\omega I)^2) \qquad [14]$$

The design curve can be obtained by setting the maximum value of y(t), i.e., where dy/dt = 0, equal to a critical displacement y_c, sometimes taken as 1/175 of the linear dimension of the component [http://en.wikipedia.org/wiki/Curtain_wall].

Design curve equations in dimensionless variables This then leads to the equations:

$$\exp(-xp/i) = (1/p) +\cos x \qquad [15]$$

$$\sin x = 1/i \qquad [16]$$

where the dimensionless variables p and i have been introduced. They are related to the actual overpressure P and impulse I by

$$p = P_0/\omega^2\chi \qquad [17]$$

$$i = I/\omega\chi \qquad [18]$$

where

$$\chi = y_c M/A \qquad [19]$$

The solution of eqs. [18] and [19] are shown in Figure 10. It was obtained by numerical solution.

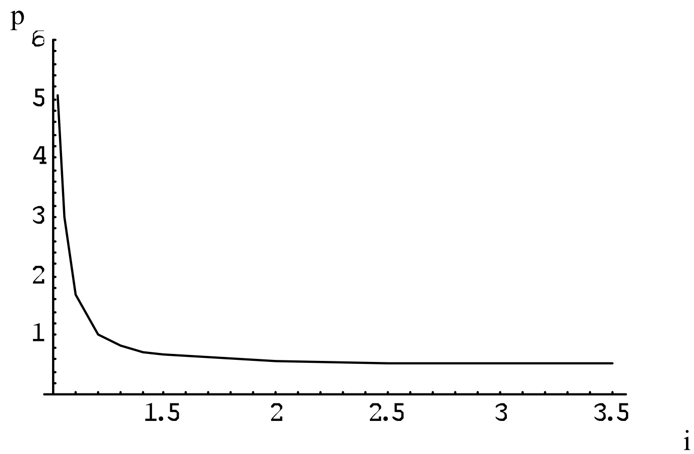


Figure 10. SDOF design curve obtained from eqs. [18] and [19] for the dimensionless variables p and i. The dimensionless variables are related to the actual overpressure and impulse by $p = P/\omega^2\chi$, $i = I/\omega\chi$, where $\chi = y_c M/A$ is related to the critical displacement, mass, and component area.

The design curve displays two asymptotes, one for p>>1 and i approaching 1, and the other for i>>1 and p approaching ½. It is easy to see from eqs. [18] and [19] that the asymptotic behaviour is described by

$$p \approx [1-(1/i^2)]^{-1/2} \qquad \text{when } p \gg 1 \qquad [20]$$

$$p \approx 1/2 + (\pi/8)(1/i) \qquad \text{when } i \gg 1 \qquad [21]$$

The design curve of Figure 10 in the dimensionless variables p and i is a universal curve: it applies whenever a structural component can be represented by a SDOF mass attached to a spring.

Design curve in actual overpressure and impulse To relate the design curves in the dimensionless variables to the design curves for an actual situation, the natural angular oscillation frequency ω of the system and the parameter χ must be known.

As an example, suppose we take the natural period of the system to be 0.11 sec. ($\omega = 56$ sec^{-1}). This is in the range of the values shown in a specific report provided by an explosives consultant. In addition, let us take the curve to pass through the design point: 8 psi and 170 psi msec.

From eqs. [20]&[21] we see that for a design point (P_d,I_d), the corresponding point (p_d,i_d) in the dimensionless space satisfy the relation

$$P_d/I_d = \omega\,(p_d/i_d) \qquad [22]$$

With the choice of the natural oscillation frequency of 56 sec^{-1} along with the design point (P_d = 8 psi and I_d = 170 psi msec = 0.17 psi sec), eq. [25] requires p_d/i_d = 0.84. From Figure 10, it can be seen that p_d = 1.01145 and i_d = 1.2. For this example, then,

$$\chi = I_d/i_d\ \omega = 0.0025 \qquad [23]$$

Accordingly

$$P = 8 \text{ psi} \qquad [24]$$

$$I = 142 \text{ psi-msec} \qquad [25]$$

This design curve is shown in Figure 11.

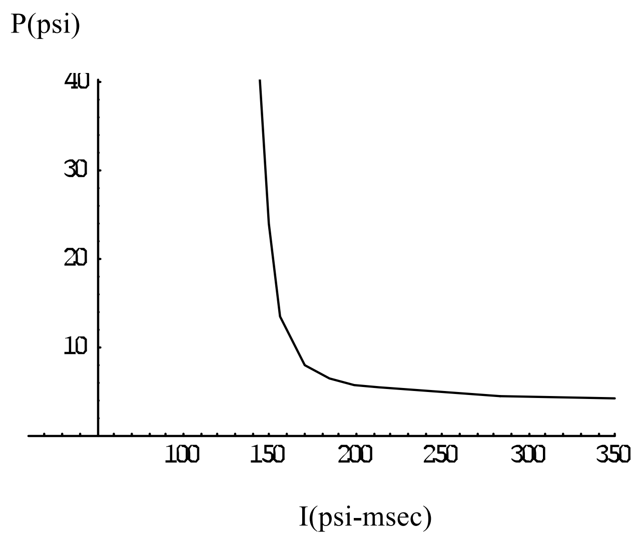


Figure 11. Design curve passing through 8 psi and 170 psi msec for a natural oscillation angular frequency of 56 sec^{-1}.

As another example, suppose instead the design point is taken to be 16 psi and 185 psi msec, and the natural oscillation angular frequency remains at 56 sec^{-1}. The same procedure leads in this case to

$$P = 9.42\ p \quad \text{psi} \qquad [26]$$
$$I = 168\ i \quad \text{psi-msec}$$

The resulting design curve for 16 psi and 185 psi msec is shown in Figure 12 (dashed) and compared to the design curve of Figure 10 for 8 psi and 170 psi msec.

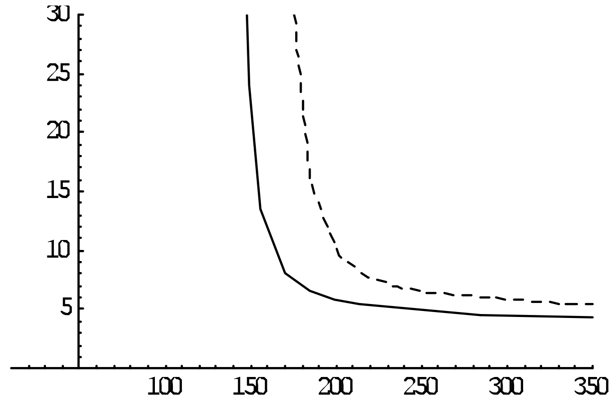


Figure 12. Design curves for blast protection. The solid curve is chosen to pass through 8 psi and 170 psi msec for a natural oscillation angular frequency of 56 sec^{-1}. The dashed curve is chosen to pass through 16 psi and 185 psi msec, with the same natural oscillation angular frequency of 56 sec^{-1}.

Design curve translated into (r,m) space The level of blast protection depends on the particular design curve chosen. In the next section, the design curves will be combined with the probability function discussed in Section 2 to obtain a confidence level for blast protection. For that purpose, it will be convenient to transform the design curves depicted in Figure 12 in the (overpressure, impulse) plane to the (equivalent TNT mass m, standoff distance r) plane. This can be done using eqs. [12] and [13]. Figure 13 displays the design curves of Figure 12 in the (m,r) plane.

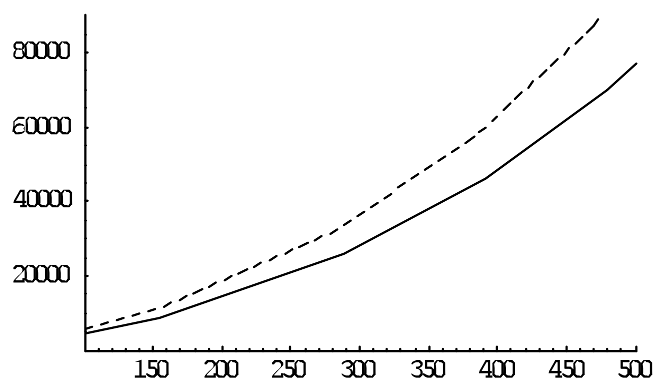


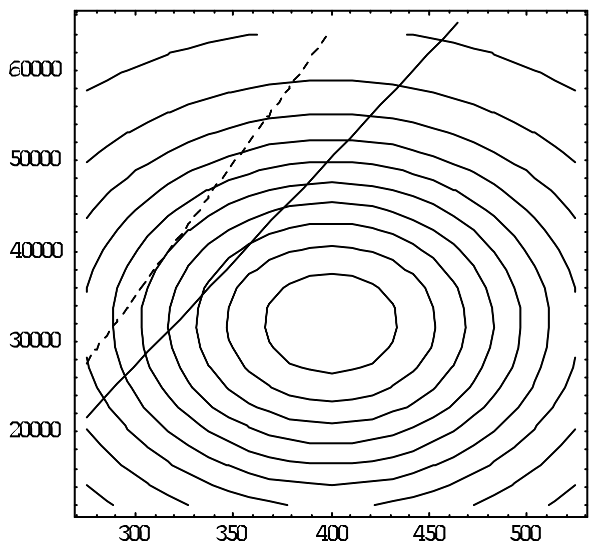
Figure 13. Design curves for blast protection. These are the same design curves as in Figure 12, but plotted here in the (m,r) plane. The solid curve is chosen to pass through 8 psi and 170 psi msec for a natural oscillation angular frequency of 56 sec^{-1}. The dashed curve is chosen to pass through 16 psi and 185 psi msec, with the same natural oscillation angular frequency of 56 sec^{-1}.

6. Probability of blast protection

The significance of a design curve in Figure 13 is that the associated structural component is protected from any blast for which the TNT equivalent mass and standoff distance gives a point lying under the curve. Accordingly, the probability that the component will withstand a blast is obtained by integrating the probability function over all points (m,r) that lie underneath the curve.

The design curves of Figure 13 are almost linear through the portion of the region they occupy where the probability function is zero. Linear fits to the curves in this portion are shown superimposed on the contour diagram for the probability function (of Figure 4), in Figure 14. The curves are shown only in the range of m and r where the probability function is nonzero.

Equivalent TNT mass (lbs)



Standoff distance (feet)

Figure 14. Design curves of Figure 13 superimposed on the probability function contour diagram of Figure 4. Again, the solid design curve is chosen to pass through 8 psi and 170 psi msec for a natural oscillation angular frequency of 56 sec^{-1}, whereas the dashed curve is chosen to pass through 16 psi and 185 psi msec, with the same natural oscillation angular frequency of 56 sec^{-1}. The probability function $F_{mr}(m,r)$ is given by eq. [5] as the product of truncated normal distributions in m and r.

For both design curves, it is seen that the maximum of the probability function lies below design curves. It is also seen that more of the probability function is below the dashed curve (16 psi, 185 psi msec design) than below the solid curve (8 psi, 170 msec). Thus, it would be expected that the confidence level for protection for the former would be higher than for the lower.

For the design curves depicted in Figure 14, the probability $P_{protected}$ that the structural component will be protected against a blast is

$$P_{protected} = \int_{r_{min}}^{r_{ma}} dr \int_{m_{min}}^{\text{smaller of } \{m_{design}(r),\, m_{max}\}} F_{mr}(m,r)\, dm \qquad [27]$$

where $m_{design}(r)$ denotes the design curve. For the two representative design curves of Figure 14, this integration gives:

$P_{protected}$ (8 psi, 170 psi msec, 56 sec^{-1} design) = 80%

$P_{protected}$(16 psi, 185 psi msec, 56 sec^{-1} design) = 91%

i.e., the probabilities of blast protection for the two designs are **80%** and **91%,** respectively.

7. Summary and discussion

7a. Summary

A probabilistic risk assessment procedure has been followed to estimate the level of blast protection at a facility. We have assumed that the damage would be collateral, with the principal terrorist target being a nearby facility. A probabilistic approach was adopted because there are uncertainties associated with both the TNT-equivalent mass of the explosive and with the standoff distance of the detonation. These uncertainties in explosive mass and standoff distance translate through commonly used scaling laws into uncertainties in the blast wave overpressures and impulses. The overpressures and impulses are of more direct interest to structural designers.

Normal distributions, with cutoffs at reasonable minimum and maximum values, have been assumed for both the TNT equivalencies of the charge and the standoff distances [Figure 3]. A structural component design curve for blast mitigation has been derived from an oft-used simple single degree of freedom (SDOF) model. This design curve defines the boundary between the overpressures and impulse combinations from which the structural component will be protected and those for which the design affords no protection. Use of the scaling laws translates this information into a boundary between combinations of TNT-equivalent mass and standoff distance. The probability of protection is then obtained by integrating the normal probability distributions over all masses and distances that lie on the protected side of the design curve.

Two numerical examples have been given, both for a structural component that has a natural oscillation period of 0.11 sec, corresponding to a natural oscillation angular frequency of 56 sec^{-1}. In the first example, the design curve has been chosen to pass through the point (8 psi, 170 psi msec); the resulting probability of protection is 80%. In the second example, the design curve has been chosen to pass through the point (16 psi, 185 psi msec): the resulting probability of protection for this case is 91%. Thus, the first example gives protection at the 1.3-sigma level, whereas the second example provides protection at the 1.7-sigma level.

7b. Discussion

This report has focused on deriving the probability of protection afforded by a specific window design as a result of an explosive blast.

It should be noted that as more information is known about risk mitigation such as the emplacement of bollards to enforce standoff, the geometry of a target facility relative to the surrounding streets, and likely screening procedures, improvements can be made on the normal truncated probability distributions assumed for TNT-equivalent masses and standoff distances.

In addition, we believe that the probabilistic approach in this report and applied to a specific terrorist-initiated explosive threat can find application in a much broader range of operational risk problems. A general four-step process is envisioned:

1. The starting point is to assume an appropriate probability function (often a normal distribution) for a set of parameters that is inherently random, and that have some (indirect) connection with the mitigation.

2. The second step is to develop equations (scaling laws) that relate these inherently random parameters to a second set of parameters that most directly impact the mitigation under consideration.

3. The third step is to identify design curves (or surfaces) for a group of selected risk-aversion measures in the second set of parameters.

4. The fourth step is to use identified scaling laws to map the design curves (surfaces) onto the space of inherently random parameters, and to integrate the normal probability function over that portion of the space that is protected by the mitigation design. This then gives the probability that the applied mitigation will be successful, i.e. it provides statistical confidence in a successful implementation.

Alternatively, steps 3 and 4 can be replaced by:

3 (alternate). The third step is to use the results of the first two steps to generate a probability distribution for the second set of parameters.

4 (alternate). The fourth step is to integrate the probability function over the portion of the space of the second set of parameters that is defined by the design curves (surfaces) for selected risk mitigation measures. This then gives the probability that the risk mitigation will be successful.

The choice between Steps 3 and 4 or the alternate Steps 3 and 4 could be based on the relative ease of calculation for the two options in a given risk scenario.

The approach can be used to systematically estimate the effectiveness of mitigation measures and associated costs required to achieve varying degrees of protection.

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