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Comparison of Window Stresses from Explosions and Projectiles

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Abstract

Blast film is traditionally applied to windows to reduce the risk of flying glass when subjected to explosion-induced stresses. However, little attention has been paid to the effect of projectiles on windows and understanding potential benefits derived from applying blast film. To that end, simple scaling laws are derived for maximum stresses in windows impacted by both explosive blasts and projectiles such as bullets or rocks. The effect of films and laminations are described, and recommendations are made for safe window designs to protect against both explosive blasts and projectiles. Comparisons are made with typical design recommendations as well as window and film properties.

1. Introduction

There is considerable interest in minimizing the effect of explosive-induced forces on glass structures such as windows. Glass fragmentation is one of the significant causes of injury and death in explosive scenarios, and many buildings incorporate some form of window treatment such as lamination, tempering, or application of security films, in order to mitigate this risk.

In addition, projectiles such as rocks and bullets that cause spatially localized and thereby highly concentrated forces upon impact, also pose risk to individuals inside facilities. It is useful, therefore, to understand the effect of standard window treatments on such threats, since these treatments are mostly used to reduce the effect of overpressures and impulses associated with explosive forces.

The purpose of this paper is to summarize a simple theoretical treatment of the problem of window breakage thresholds for both explosive and projectile scenarios. Uncomplicated scaling laws are presented for these thresholds.

Section 2 summarizes the basic analytic approach: the use of a dynamic equation for the distortion of a plate in response to an applied stress, and its solution by a variational principle.

Section 3 describes the maximum stresses developed in an uncoated window of uniform thickness both for an explosive blast and for a projectile.

Section 4 gives the modifications in the results when the window has one or more films or laminations.

Section 5 summarizes the results and compares them with typical design guidelines, window properties, and film properties.

The results are discussed briefly in Section 6.

For ease of reference, the typical design guidelines and window and film properties are summarized in the Appendix.

2. Summary of analytic approach

It is not surprising that a window responds differently to an explosive blast than to the impact of a projectile. The blast creates a force over the entire surface of the window whereas the force exerted by a projectile is localized to a small area.

To estimate the breakage thresholds from a blast or projectile, we use an equation that describes the response of a thin solid (a plate) to applied forces.

For a single uniform pane of glass, the dynamic plate equation is

$$(\nabla_x^2 + \nabla_y^2)(\nabla_x^2 + \nabla_y^2)\, w - (\rho h/D)\partial^2 w/\partial t^2 = P/D \qquad [1]$$

where

w is the displacement perpendicular to the (x,y) plane of the plate
h is the thickness of the plate
ρ is the mass density of the plate
P(x,y, t) is the pressure exerted on the plate

and

$$D = Yh^3/[12(1-\upsilon^2)] \qquad [2]$$

Here,

Y is Young's modulus of the plate
υ is the Poisson ratio of the plate.

Equation [1] is obtained from the familiar static plate equation as given, e.g., in Timoshenko (1940), by adding the term $(\rho h/D)\partial^2 w/\partial t^2$ to account for the inertial effects associated with acceleration.

Although eqs. [1] and [2] only apply to a uniform (i.e., non-laminated) situation, we shall see in Section 4 below that it is straightforward to modify the parameters so as to make eq. [1] applicable to a laminated case as well.

The plate equation is fourth order in the spatial derivatives (in the plane of the plate) and second order in the time derivative. Because it is a linear equation in w, it is useful to Fourier analyze it both in time and space. This is tantamount to describing the motion as a weighted sum over the normal modes of the plate, where the weighting is determined by the time and spatial dependence of the applied pressure P(x,y,t).

For example, when the applied pressure is due to an explosive blast that exerts a uniform pressure over the entire surface of the window, the normal mode that corresponds to the most uniform displacement of the window dominates, with the weighting for the higher normal modes that describe greater spatial variation being much less. When the applied pressure is due to a projectile, again the normal mode that corresponds to the most uniform displacement dominates, since the window experiences a net overall displacement. However, although the weighting for the other normal modes again drops off rapidly, the drop-off is not quite as rapid as for the blast case.

For a window of arbitrary shape, it is useful to employ an integral variational principle form of the plate equation rather than the differential expression of eq. [1] directly. This is because the variational principle form is less sensitive to the exact form of the solution, so that approximate expressions can be used for the displacement w without compromising the results.

To obtain the variational principle, the displacement w(x,y,t) is first Fourier analyzed in the time variable. This introduces the normal mode angular frequency ω. There should exist a Lagrangian for which eq. [1] is the Euler-Lagrange equation. Since eq. [1] is fourth order in the spatial derivatives, it would be expected that the Lagrangian contains second order derivatives.

It is easy to verify that

$$\delta[\omega^2\rho h/D] = 0 \qquad [3]$$

where

$$\omega^2\rho h/D = \iint dxdy\ [\iint(\partial^2 w/\ \partial x^2 + \partial^2 w/\ \partial y^2)^2]\ /\ \iint dxdy w^2 \qquad [4]$$

gives eq. [1] as its Euler-Lagrange equation. [See, for example, Courant & Hilbert (1953).]

Equations [3] and [4] are the equations that are used to obtain the results in the following sections, using trial functions for w that are products of the exact normal modes for a 1-dimensional window (where a 1-dimensional window is (a fictitious) one in which w depends only on a single spatial variable).

3. Stresses in a uniform clamped rectangular window with no film

The maximum stresses developed in a window with no security film are different for an explosive blast than for a projectile. This is related directly to the more spatially uniform response of the window to a blast than to a projectile.

For a blast, the components of the maximum stress, which occurs at the center of the window, are found to be:

$$\sigma_x(x=y=0) \approx \{Y^{1/2}PT(L_y/L_x)(1-\nu^2)^{1/2}/(h\rho^{1/2})\}[4^5/(3^{7/2}\pi^4)] \qquad \textbf{Blast} \qquad [5]$$

$$\sigma_y(x=y=0) \approx \{Y^{1/2}PT(L_x/L_y)(1-\nu^2)^{1/2}/(h\rho^{1/2})\}[4^5/(3^{7/2}\pi^4)] \qquad \textbf{Blast} \qquad [6]$$

Here the subscripts on the stress refer to the component (direction) of the stress, PT is the impulse per unit area exerted on the window by the blast, L_x is the overall dimension of the window in the x-direction, L_y is the overall dimension of the window in the y-direction, and the other symbols are as defined earlier.

Note that

a. The stresses are independent of the size of the window, depending only on the ratio of the dimensions in the x and y directions. (The independence is because the stress is proportional

to the second derivative of the displacement; and the displacement itself is proportional to the square of the window dimension whereas $\partial^2/\partial x^2$ and $\partial^2/\partial y^2$ are each inversely proportional to the square of the window dimension.)

b. The stresses depend inversely on the thickness of the window, so that thicker windows will have less stress.

c. Interestingly, the stresses are proportional to $(Y/\rho)^{1/2}$, the square root of the Young's modulus divided by the density.

d. The stresses are also proportional to the specific impulse, PT.

The stresses would also be expected to be large at the window edges, especially at the midpoints of the edges. Evaluation of the expressions at those positions show the stresses are comparable to that at x=y=0.

For a projectile impacting the window at its center – i.e. at its most vulnerable point, the maximum stress is found to be:

$$\sigma_x\,(\max) \approx \{Y^{1/2}MV(1-\nu^2)^{1/2}/(h\rho^{1/2})\}[8x3^{1/2}/3\,\pi^2)] \qquad \textbf{Projectile} \qquad [7]$$

$$\sigma_y\,(\max) \approx \{Y^{1/2}MV(1-\nu^2)^{1/2}/(h\rho^{1/2})\}[8x3^{1/2}/3\,\pi^2)] \qquad \textbf{Projectile} \qquad [8]$$

Here, M_1 is the projectile mass per unit area of impact and V is the projectile speed.

As with the blast case,

a. The stress is proportional to $(Y/\rho)^{1/2}$, the square root of the Young's modulus divided by the density.

b. The stress is inversely proportional to the window thickness.

It is also interesting to note that:

c. The dimensions of the window do not enter. This might be expected from the localized impact of the projectile.

d. The stress is proportional to the initial momentum M_1V. Note, however, that M_1 is the mass of the projectile *per unit area of the impact*. This shows that a pointed bullet, or a pointed spear can be more effective in causing damage than a blunt projectile of the same mass.

The magnitude of the numerical factor in the square bracket for the blast case is **0.225**, and the magnitude of the numerical factor in the projectile case is **0.47**.

Thus, we note a two-fold increase in the stress for a given impulse per-unit-area for the projectile compared to the blast case. This is due to the difference in the amplitudes of the normal modes excited in the two cases.

4. Stresses in a window with coatings or laminations

The results of the previous section can be modified straightforwardly to describe the effect of laminations or films applied to windows. When a plate is bent, the internal stresses set up inner moments which, with the moments from the inertial forces, counter the external moments on the plate due to the applied pressures. For a plate without laminations, the internal stress at any position (x,y,z) in the plate is directly proportional to the bending curvature, with the proportionality constant depending on the Young's modulus Y, Poisson ratio υ, and the bending moment arm (z-h/2).

When laminations are present, the moment arm is changed, and Y and υ also depend on z. Similarly, the density ρ also depends on z, so that the moment of the inertial force needs to take that into account. The net result is that for a laminated window (or a window with one or more layers of film), the earlier equations are modified by simply making the replacements

$$\rho \rightarrow \Sigma_i (h_i / h) \rho_i \qquad [9]$$

$$D \rightarrow \Sigma_i h_i [Y_i / (1-\upsilon_i^2)](z_i - z_N)^2 \qquad [10]$$

where

$$z_N = \Sigma_i h_i z_i [Y_i/(1-\upsilon_i^2)] / \{ \Sigma_i h_i [Y_i/(1-\upsilon_i^2)]\} \qquad [11]$$

Here the subscript i refers to the ith layer in the window, and the summation is over all of the layers (laminations, films)

Our primary focus in this paper is on the ability of a film to hold a shattered window together. In that case, the theoretical treatment is simplified considerably.

Specifically, once the glass is fragmented, to the first approximation we can assume that it will not contribute to the internal bending moments: After it fragments, it only contributes to the mass per unit area. The internal bending moments are then all due to the tension forces in the film.

Accordingly, in the expressions of Section 3, let all the quantities refer to the film alone, except for the density. The density will now be changed to

$$\rho_{effective} \approx \rho_{glass} (h_{glass}/h_{film}) \qquad [12]$$

To reiterate, the rationale for this substitution is that the film now supports all of the inertia and applied forces in the system, with the glass no longer contributing to the internal bending moments. The calculated stress can then be compared to the tensile strength of the film.

5. Comparison with typical windows and films

Approximate scaling laws have been derived for the maximum stresses that can be expected in windows subjected to explosions and to projectiles. These have resulted from a variational principle based on a dynamic plate equation. The effect of lamination and films has been included by modifying the parameters appearing in the plate equation.

In this section, the scaling laws of Sections 3 and 4 are compared with actual data. For ease of reference, this data is collected in the Appendix.

From the open literature data recorded in the Appendix, it is found that

1. The Young's modulus for the glass is 20X larger than that for the film.
2. The tensile strength of the film is 5X larger than that for the glass.
3. A typical thickness of film (0.15 mm) is only 1/50 that of the glass (7.5 mm)

5a. Explosive blasts

Blast: glass window without film

From the data in the Appendix, consider the following situation:

Blast overpressure	(2.8 – 27.6) x 10^5 dynes/cm^2,
Duration	10^{-2} seconds
Equal x and y dimensions	
Glass thickness	0.75 cm
Glass density	2.5 g/cc
Young's modulus	6.9 x 10^{11} dynes/cm^2
Poisson ratio	0.23

Then eq. [5] gives for the resulting maximum stress at the center of the window:

$$\sigma(\max) \approx (4.3 - 42.3) \times 10^8 \text{ dynes/cm}^2$$

This is to be compared with the maximum tensile strength of the glass:

$$\text{Tensile strength} = (4.1\text{-}4.5) \times 10^8 \text{ dynes/cm}^2.$$

Accordingly, we find that for ordinary blast overpressures of 2.8 x 10^5 dynes/cm^2, the window is at the limit of withstanding the blast. However, with respect to the government-recommended specification to withstand a pressure of 27.6 x10^5 dynes/cm^2, the window stress exceeds the tensile strength by about a factor of ten.

Blast: glass window plus film

Next consider the case where the stress induced by the explosion is so large that it exceeds the tensile strength of the glass, and the glass shatters. In that case, only the film can hold the window plus film system together. The relevant parameters for this case are therefore:

Blast overpressure	$(2.8 - 27.6) \times 10^5$ dynes/cm^2
Duration	10^{-2} seconds
Equal x and y dimensions	
Glass thickness	0.75 cm
Glass density	2.5 g/cc
Film thickness	0.015 cm
Film Young's modulus	3.45×10^{10} dynes/cm^2
Poisson ratio	~ 0.33
Film tensile strength	2.08×10^9 dynes/cm^2

Inserting these parameters into eq. [5] and using the effective density just derived, we find

$$\sigma(\max) \approx (0.63 - 6.3) \times 10^9 \text{ dynes/cm}^2$$

This is to be compared with the film tensile strength

$$\text{Film tensile strength} \approx 2.08 \times 10^9 \text{ dynes/cm}^2$$

Interestingly, for a normal overpressure of 2.8×10^5 dynes/cm^2, the film tensile strength is about 3X the maximum induced stress, whereas for the government-recommended overpressure limit of 27.6×10^5 dynes/cm^2, the induced stress exceeds the film tensile strength by a factor of about three.

The film has indeed improved the situation.

The maximum induced stress can be decreased by adding more layers of film. Unfortunately, eq. [5] shows that with the effective mass density, the maximum induced stress is only proportional to $h_{film}^{1/2}$. Thus if four layers of film are used, the maximum induced stress is reduced by only a factor of two.

Our equations suggest that the solution for increased blast resistance is to use glass that has been heat treated to obtain a greater tensile strength, in combination with multiple layers of film.

For example, if the heat treated glass has a tensile strength that is larger than that of untreated glass by a factor of three, then that glass could be used with a single film to make a system that would satisfy the government's most stringent case.

5b. Projectiles

The relevant equation here is eq. [7].

Projectiles on glass alone

Consider the situation described in the Appendix for tempered glass:

Breaking stress (60 sec load)	16.6×10^8 dyne/cm^2
Impact velocity	1829 cm/s
Mass of missile	5 gm
Area of impact	0.4 cm^2
Glass thickness	0.75 cm
Glass density	2.5 g/cc
Young's modulus	6.9×10^{11} dynes/cm^2
Poisson ratio	0.23

For these parameters, eq. [G4] gives for the maximum induced stress:

$$\sigma \text{ (max)} \approx 7.25 \times 10^9 \text{ dynes/cm}^2$$

This is to be compared with the tensile strength of the tempered glass:

$$\text{Tensile strength} = 1.7 \times 10^9 \text{ dynes/cm}^2$$

The maximum induced stress exceeds the tensile strength by a factor of 4.3. Accordingly, the projectile would shatter the glass and penetrate.

Projectiles on glass plus film

Next suppose that a thin film is added to the glass. For the tempered glass, the tensile strength of 1.7×10^9 dynes/cm^2 is comparable to the tensile strength of the film (e.g. 2.08×10^9 dynes/cm^2). However, the film's Young's modulus is 20X smaller than that of the glass, so the maximum stress developed in the film could be smaller.

To see if this is the case, again consider a situation where the glass has shattered, and the only contribution to the induced moments is that due to the film. As before, the glass now contributes to the problem only through its effect on the mass density:

The relevant parameters are:

Glass thickness	0.75 cm
Glass density	2.5 g/cc
Film thickness	0.015 cm
Film Young's modulus	3.45×10^{10} dynes/cm^2
Poisson ratio	~ 0.33
Film tensile strength	2.08×10^9 dynes/cm^2

With all the parameters in eq. [7] now being those of the film – except for the effective mass density, which is given by eq. [12], eq. [7] gives for these parameters:

$$\sigma \text{ (max)} \approx 4.47 \times 10^9 \text{ dynes/cm}^2$$

This is to be compared with the tensile strength of the film:

$$\text{Tensile strength} = 2.08 \times 10^9 \text{ dynes/cm}^2$$

The maximum induced stress in the film is 2.2X the tensile strength of the film, whereas for the glass by itself, the maximum induced stress is 4.3X the tensile strength of the glass.

To summarize, the theoretical predictions give results that are within a factor of five of the data recorded in the Appendix.

6. Recommendations

The simple scaling laws and the data comparisons of the previous section give support to the following recommendations:

For increased blast resistance only:

1. Use glass that has been heat treated to obtain a greater tensile strength.
2. Additionally strengthen the glass with one or more layers of film.

For example, if the heat treated glass has a tensile strength that is larger than that of untreated glass by a factor of three, then that glass could be used with a single film to make a system that would satisfy the government's most stringent case.

For both blast and projectile-resistance:

1. Use a plastic-like material that has a relatively low Young's modulus (like the film material) but that has a thickness similar to a standard glass window.
2. Stiffen by adding layers of tempered glass (with a larger Young's modulus) of comparable or less thickness.

The film and tempered glass would both have tensile strengths of the same order of magnitude, but the lower effective Young's modulus for the system would result in a smaller induced stress.

The foregoing treatment has been somewhat crude: Nonlinear effects have not been taken into account, and no attempt has been made to describe the interesting radial and concentric fracture patterns that can result when a window breaks. Nevertheless, the results are within a factor of at most five of the guidelines in the Appendix, and so it is felt that the resulting simple scaling laws of eqs. [5]-[12] can be useful as a rough analytic guide to designing window systems that are resistant to both blasts and projectiles.

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Appendix

This appendix summarizes typical design guidelines and properties of window glass and films.

Typical design guidelines for blasts [from the Architectural Record accessible on line at http://archrecord.construction.com/print.asp? (2006)]

Common blast level: 4 psi overpressure at an impulse level of 28 psi x milliseconds

Enhanced blast level: 10 psi overpressure at an impulse level of 89 psi x milliseconds

Some government agency requirement: 40+psi overpressure for a blast duration of several hundred milliseconds

Note that 1 psi = 6.9×10^4 dynes/cm^2. Accordingly, in cgs units, the above translates to

	Overpressure (dynes/cm^2)	Duration (sec)
Common	2.8×10^5	7×10^{-3}
Enhanced	6.9×10^5	8.9×10^{-3}

Window glass properties [from Table 1-91 of the CRC Handbook of Tables for Applied Engineering Science, 2nd Edition, Ed. Ray E. Bolz and George L. Tuve, Boca Raton, Fl: CRC Press, Inc. (1986)]

Window glass specifications satisfying Federal Specification Standard DD-G-451c

Density	2.5 g/cc
Young's modulus	6.9×10^{11} dynes/cm^2
Poisson ratio	0.23
Tensile strength	4.1-4.5 x 10^8 dynes/cm^2

Typical security window film properties [3M Scotchshield™ Ultra Safety and Security Window Films from the 3M website www.3M.com/window/film (2011)]

	SCLARL150	Ultra 400 Series	Ultra 600
Film thickness	0.051 mm	0.1 mm	0.152 mm
Young's modulus	$>3.45 \times 10^{10}$ d/cm^2	$>3.45 \times 10^{10}$ d/cm^2	$>3.45 \times 10^{10}$ d/cm^2
Tensile strength	2.08×10^9 d/cm^2	2.08×10^9 d/cm^2	2.08×10^9 d/cm^2

Typical recommendations for explosion protection anti-shatter film [from Applied Products, http://www.appliedproducts.co.uk/index.php?cpage=fullstory&article id=90&item type=SEC (2011); See also Carl S. Young, Metrics and Methods for Security Risk Management, Burlington, MA, Syngress (2010)]

"Polyester film at least 175 microns (0.175 mm) should be used: 300 micron (0.3 mm) film should be considered for panes over 10 square meters or for ground floor windows over 3 square meters."

"The specification can be lowered to at least 100 microns (0.1 mm) if bomb blast net curtains are also to be used."

Recommendations of the British Security Service MI5 (2006)

The film specifications that Applied Products gives are those cited in the MI5 recommendations for anti-shatter film in its document for "Protection against flying glass" [www.mi5.gov.uk/output/Page169.html (2006)]

In addition MI5 recommends the use of blast resistant glass (i.e., laminated glass) with the following specifications:

Minimum thickness:	7.5 mm
Inclusion of polyvinylbutryal interlayer of min. thickness	1.5 mm
Frame mounting able to withstand	7×10^4 dynes/cm^2

These specifications apply to a window pane with area <2 m^2. A 1 square meter window has an increased blast resistance, so that the numbers should be increased by 50% to match the increased resistance.

For larger windows, the recommendation is that the 7×10^4 dynes/cm^2 should not be decreased when designing the accompanying frames.

Tempered glass properties [http://www.alumaxbath.com/tech/tgp.htm (2004)]

In the production of regular glass, a molten silica-based mix is cooled slowly under carefully controlled conditions. The slow cooling (annealing) relieves undesirable stresses from the glass. Increased strength can be obtained by heating the annealed glass to a temperature near its softening point and then cooling it rapidly. The resulting heat-treated glass is classified either as "fully tempered" or "heat-strengthened".

Typical breaking stresses and impact velocities for fracture are as follows:

	Annealed glass	Tempered glass
Breaking stress (60 sec load)	4.14×10^8 dyne/cm^2	16.6×10^8 dyne/cm^2
Impact velocity (1/4" 5 g missile)	914 cm/s	1829 cm/s